TEMPERATURE FIELD OF A PLATE WITH SLOTTED FILTRATION

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An analytical solution is given for the problem of the steady temperature field of a homogeneous plate dividing two media having different temperatures and having a continuous plane slot, through which one of the media is forced.

In connection with the wide development of panel construction in the housing market, it is important to investigate the temperature field of the zone of coupling of two adjacent panels. This zone, as a rule, contains an air gap (a slot), along which air can diffuse; this diffusion has a considerable effect on the temperature field of the panel and, hence, on the thermal regime of the room.

The available analytical [1, 2] and numerical [3, 4] investigations of the problem under consideration so far have not yielded a sufficiently simple and universal calculational relationship for the temperature field of a plate with slotted diffusion.

The problem consists of finding a plane, with temperature field of a plate dividing two media: an inner medium with temperature t_i and an outer medium with temperature t_o. The heat-exchange coefficients on the respective plate surfaces equal α_i and α_o. The plate has a continuous plane slot of width h, along which with velocity v there moves an infiltrant, with known density ρ_0, specific heat c_0, viscosity ν_0, and thermal conductivity λ_0. Also given are the plate thickness δ, the plate thermal conductivity λ, and the heat-exchange coefficient of the infiltrant in the slot α_0.

To simplify the solution of the problem, we approximate third-order boundary conditions on the surfaces of the plate by boundary conditions of the first kind, after replacing the given plate by an equivalent plate of thickness $\delta_1 = \lambda/\alpha_o + \delta + \lambda/\alpha_i$ with given temperatures t_o and t_i of the outer and inner surfaces of the equivalent plate.

We superpose the axis of abscissas on the surface of the slot, and the axis of ordinates on the outer surface of the equivalent plate, having chosen the quantity δ_1 as the scale of the dimensionless coordinates x and y.

Proceeding from the symmetry of the temperature field with respect to the slot axis, we write the heat-balance equations on the walls of the slot, connecting the temperature of the infiltrant $t_0(x)$ with the temperature of the plate $t(x, y)$, in the form

$$\frac{\rho_0 c_0 v h}{2\delta_1}\,\frac{dt_0(x)}{dx} = \alpha_0\,[t(x,\ 0) - t_0(x)] = \frac{\lambda}{\delta_1}\,\frac{\partial t(x,\ 0)}{\partial y} \qquad (1)$$

or, converting to the dimensionless temperatures

$$\theta_0(x) = \frac{t_0(x) - t_o}{t_i - t_o}, \quad \theta(x,\ y) = \frac{t(x,\ y) - t_o}{t_i - t_o}$$

and the dimensionless groups

$$\mathrm{Nu} = \frac{\alpha_0 h}{\lambda_0}, \quad \mathrm{Re} = \frac{vh}{\nu_0}, \quad \mathrm{Pe} = \frac{\rho_0 c_0 v h}{\lambda_0}, \quad \mathrm{Bi} = \frac{\alpha_i \delta_1}{\lambda},$$

$$\mathrm{Gz} = \mathrm{Pe}\,\frac{h}{\delta_1}, \quad k = \frac{\alpha_o}{\alpha_i}, \quad m = \frac{\delta_1}{h}, \quad \beta = 2\,\frac{\mathrm{Nu}}{\mathrm{Gz}}, \quad p = \frac{\lambda}{\lambda_0}, \quad s = \frac{\alpha_0}{\alpha_i},$$

we can write system (1) in the form of two equations:

$$\frac{d\theta_0}{dx} = \beta\,[\theta(x,\ 0) - \theta_0], \qquad (2)$$

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$$\theta(x, 0) = \theta_0(x) + \frac{p}{m\,\mathrm{Nu}} \frac{\partial\theta(x, 0)}{\partial y}. \tag{3}$$

In this case, for a plane slot, with linear variation in wall temperature, as is known [5, 6], for laminar flow (Re < 10^3) we have

$$\mathrm{Nu} = 4.12 \quad (\mathrm{Gz} < 20), \quad \mathrm{Nu} = 1.47\mathrm{Gz}^{1/3} \quad (\mathrm{Gz} > 20), \tag{4}$$

i.e.,

$$\beta = 8.24\mathrm{Gz}^{-1} \quad (\mathrm{Gz} < 20), \quad \beta = 2.94\mathrm{Gz}^{-2/3} \quad (\mathrm{Gz} > 20). \tag{5}$$

Solving Eq. (2) with condition $\theta_0(0) = 0$ (equating the temperature of the infiltrant at the slot inlet to the temperature t_0 of the medium, which is subjected to infiltration), we obtain

$$\theta_0(x) = \beta \int_0^x \theta(\tau, 0) \exp[-\beta(x-\tau)]\, d\tau. \tag{6}$$

Substituting the value found for (6) into the right side of Eq. (3) we obtain a boundary condition for the unknown function $\theta(x, y)$ for $y = 0$:

$$\theta(x, 0) = \beta \int_0^x \theta(\tau, 0) \exp[-\beta(x-\tau)]\, d\tau + \frac{p}{m\,\mathrm{Nu}} \frac{\partial\theta(x, 0)}{\partial y}. \tag{7}$$

As for the other boundary condition along the y coordinate, the condition will be that the temperature field of the plate is one-dimensional far from the slot

$$\partial\theta(x, \infty)/\partial y = 0. \tag{8}$$

Finally, the boundary conditions along x will be the equality of the temperatures of the surfaces of the equivalent plate to the given temperatures t_0 and t_1, i.e.,

$$\theta(0) = 0, \quad \theta(1) = 1. \tag{9}$$

Using the Fourier method, as solution of the Laplace equation

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0, \tag{10}$$

satisfying conditions (8)- (9), we obtain

$$\theta(x, y) = x + \sum_{n=1}^{\infty} C_n \exp(-n\pi y) \sin n\pi x. \tag{11}$$

Using the remaining boundary condition (7) for determining C_n we obtain

$$C_n = \left[\frac{n\pi[1-(-1)^n \exp(-\beta)]}{\beta^2 + n^2\pi^2} - \frac{1-(-1)^n}{n\pi}\right] \Bigg/ \left[\beta\left\{\frac{1}{2}\left(1 + \frac{n\pi p}{m\,\mathrm{Nu}}\right) - \frac{n^2\pi^2\beta[1-(-1)^n \exp(-\beta)]}{(\beta^2+n^2\pi^2)^2} - \frac{\beta^2}{2(\beta^2+n^2\pi^2)}\right\}\right]. \tag{12}$$

Thus, Eqs. (11)-(12) represent the unknown solution for the temperature field of the equivalent plate. In order to convert to the temperature field of the real plate, it is necessary only to take into account that the outer surface of this plate corresponds to the coordinate $x = \lambda/\alpha_0\delta_1 = 1/k\mathrm{Bi}$, and the inner surface corresponds to the coordinate $x = 1/\delta_1(\lambda/\alpha_0 + \delta) = 1 - 1/\mathrm{Bi}$.

Finally, for the temperature of the infiltrant, from Eqs. (6) and (11) we have

$$\theta_0(x) = \beta \int_0^x \left(\tau + \sum_{n=1}^{\infty} C_n \sin n\pi\tau\right) \exp[-\beta(x-\tau)]\, d\tau. \tag{13}$$

Of greatest practical interest is the extreme temperature of that surface of the plate in the direction in which the slotted filtration occurs, i.e., in the considered case, the extreme temperature of the inner surface is equal to

$$\theta_{\mathrm{is}}(0) = \theta\left(1 - \frac{1}{\mathrm{Bi}}, 0\right) = \sum_{n=1}^{\infty} C_n \sin n\pi \left(1 - \frac{1}{\mathrm{Bi}}\right) + 1 - \mathrm{Bi}^{-1}. \tag{14}$$

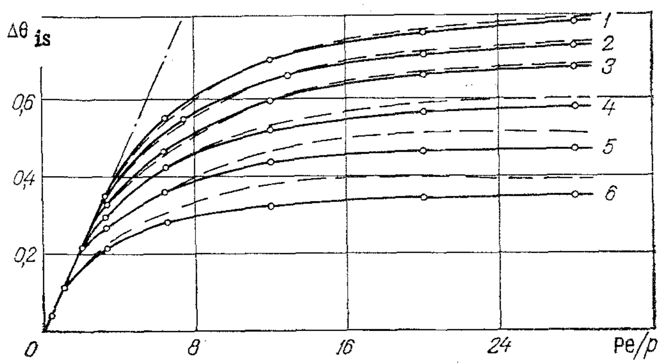

Fig. 1. Effect of filtration on extreme temperature of plate surface: solid curves) exact solution (Bi = 8.875, p = 15); dashed curves) calculation based on Eq. (20); dot–dash line) calculation based on Eq. (17): 1) sBi = 150; 2) 75; 3) 37.5; 4) 18.8; 5) 9.4; 6) 4.7.

For convenience in analyzing the obtained solution, we convert from the dimensionless temperature $\theta_{is}(0) = (t_{is}(0) - t_o)/(t_i - t_o)$ to the dimensionless temperature

$$\Delta\theta_{is} = \frac{\theta_{is}(\infty) - \theta_{is}(0)}{\theta_{is}(\infty)} = \frac{t_{is}(\infty) - t_{is}(0)}{t_{is}(\infty) - t_o} = \frac{-\mathrm{Bi}}{\mathrm{Bi}-1} \sum_{n=1}^{\infty} C_n \sin n\pi \left(1 - \frac{1}{\mathrm{Bi}}\right), \tag{15}$$

where $\theta_{is}(\infty) = 1 - 1/\mathrm{Bi}$ is the value of the dimensionless temperature of the inner surface of the plate for $y \to \infty$.

For small flow rates of the infiltrant, when Pe → 0, Gz → 0, and hence according to (5), $\beta \to \infty$, from Eqs. (12) and (15) we find

$$\Delta\theta_{is} = \frac{2\mathrm{BiPe}}{\pi^2 p(\mathrm{Bi}-1)} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\pi\left(1 - \frac{1}{\mathrm{Bi}}\right)}{(2n-1)^2} =$$

$$= \frac{2\mathrm{Bi\,Pe}}{\pi^2 p(\mathrm{Bi}-1)} \int_0^{\pi/2\mathrm{Bi}} \ln \operatorname{tg} \tau d\tau = \frac{2\mathrm{Bi\,Pe}}{\pi^2 p(\mathrm{Bi}-1)} \left[-\frac{\pi}{2\mathrm{Bi}} \ln \frac{\pi}{2\mathrm{Bi}} + \frac{\pi}{2\mathrm{Bi}} - \frac{1}{9}\left(\frac{\pi}{2\mathrm{Bi}}\right)^3 - \frac{7}{450}\left(\frac{\pi}{2\mathrm{Bi}}\right)^5 - \ldots \right]. \tag{16}$$

For the practical applications indicated above, the values 7 < Bi < 15 are characteristic; therefore, the error connected with discarding terms of the series (16) of third and higher degree does not exceed 1% of the quantity $\Delta\theta_{is}$. Therefore, as a calculational formula for small values of Pe we have

$$\Delta\theta_{is} = \frac{\mathrm{Pe}}{\pi p(\mathrm{Bi}-1)} \left(1 - \ln \frac{\pi}{2\mathrm{Bi}}\right). \tag{17}$$

Figure 1 gives a comparison of the quantities $\Delta\theta_{is}$ found analytically and numerically; the function (17) is shown by the dot–dash line.

For large values of Pe, when Gz → ∞ and $\beta \to 0$, from Eqs. (12) and (15) we obtain

$$\Delta\theta_{is} = -\frac{2\mathrm{Bi}}{\pi(\mathrm{Bi}-1)} \sum_{n=1}^{\infty} \frac{(-1)^n \sin n\pi\left(1 - \frac{1}{\mathrm{Bi}}\right)}{n\left(1 + \frac{n}{s\mathrm{Bi}}\right)}, \tag{18}$$

where for the case $s \to \infty$, Eq. (18) gives the obvious result: $\Delta\theta_{is} = 1$.

If we approximate (18) by the simpler relation

$$\Delta\theta_{is} = 1 - (s\mathrm{Bi})^{-1/3}, \tag{19}$$

then as a calculational formula, applicable for all flow rates of the infiltrant and satisfying the limiting relations (17) and (19), we have

$$\Delta\theta_{is} = [1 - s\,\mathrm{Bi})^{-1/3}]\left\{1 - \exp\left[-\frac{\left(1 - \ln\frac{\pi}{2\mathrm{Bi}}\right)}{\pi(\mathrm{Bi} - 1)(1 - (s\,\mathrm{Bi})^{-1/3})}\,\frac{\mathrm{Pe}}{p}\right]\right\}. \quad (20)$$

The values of $\Delta\theta_{is}$ calculated from Eq. (20) are shown in Fig. 1 by the dashed curves. For the practically useable region of values $h < 2$ mm, corresponding to values $s\mathrm{Bi} \equiv (m/p)\cdot \mathrm{Nu} > 35$, Eq. (20) has an error on the order of 1%, and it can be used for practical calculations.

NOTATION

t_i and t_o, air temperatures outside the plate (for the inner and outer media, respectively); $t_o(x)$, air temperature in the slot; $t(x, y)$, temperature of the plate; $\theta_o(x)$ and $\theta(x, y)$, dimensionless temperatures; $\Delta\theta_{is}$, dimensionless extreme temperature of the plate surface; $p = \lambda/\lambda_o$, ratio of the thermal conductivities of the plate and the air; $\mathrm{Pe} = \rho_o c_o v h/\lambda_o$, Peclet number; h, width of the slot; v, mean air velocity in the slot.

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MASS TRANSFER WITH A "MEMORY" WHEN DESCRIBING SORPTION PROCESSES

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The possibility of describing sorption processes by an integrodifferential equation of mass transfer with a "memory" is pointed out. An analytical solution of this equation is given.

When deriving the equations of sorption kinetics, the assumption that the isotherm has the form $a = f(c)$ is equivalent to the assumption that the act of adsorption is instantaneous [1, 2]. In fact, in many processes in which chemosorption plays an important part, the characteristic times of the acts of adsorption may be so great that they cannot be neglected. In order to take into account the inertia of the adsorption processes, we can consider an isotherm of the form

$$a = \gamma c + \int_0^\tau h(\Theta)\, c(\tau - \Theta)\, d\Theta. \quad (1)$$

In this expression the first term defines the contribution of adsorption processes with small characteristic times, while the second relates to slow adsorption processes. The general equation of the kinetics of isothermal sorption

$$\frac{\partial}{\partial\tau}(c + a) = -\,\mathrm{div}\,\mathbf{q}_m \quad (2)$$

can be represented, using Eq. (1), in the form

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